SOLUTION BY THOMAS SPENCER, SOUTH MERIDEN, CONN.

From Trigonometry we have the expansion

$$\frac{\eta \sin x}{1-2\eta \cos x+\eta^2} = \eta \sin x + \eta^2 \sin 2x + \eta^3 \sin 3x + \&c.$$

Multiply both sides of this equation by $2dx$, and integrate, we have

$$\log(1-2\eta\cos x+\eta^2) = -2\eta\cos x - \tfrac{1}{2}\,2\eta^2\cos 2x - \tfrac{1}{3}\,2\eta^3\cos 3x - \&c.$$

Also we know that

$$\log(1+\eta^2) = \eta^2 - \tfrac{1}{2}\eta^4 + \tfrac{1}{3}\eta^6 - \&c.$$

Therefore we have

$$\log\left(1-\frac{2\eta}{1+\eta^2}\cos x\right) = \log(1-2\eta\cos x+\eta^2) - \log(1+\eta^2)$$

$$= -\eta^2 + \tfrac{1}{2}\eta^4 - \tfrac{1}{3}\eta^6 + \&c.$$

$$-2\eta\cos x - \tfrac{1}{2}\,2\eta^2\cos 2x - \tfrac{1}{3}\,2\eta^3\cos 3x - \&c.$$

$$= \sum_{i=1}^{i=\infty}(-1)^i\frac{\eta^{2i}}{i} - \sum_{i=1}^{i=\infty}\frac{2\eta^i}{i}\cos ix.$$

SOLUTION BY H. HEATON.

Because $2\cos x = e^{x\sqrt{-1}} + e^{-x\sqrt{-1}}$; therefore

$$\log\left(1-\frac{2\eta}{1+\eta^2}\cos x\right) = \log\left[1+\eta^2-\eta(e^{x\sqrt{-1}}+e^{-x\sqrt{-1}})\right] - \log(1+\eta^2)$$

$$= -\log(1+\eta^2) + \log(1-\eta e^{x\sqrt{-1}}) + \log(1-\eta e^{-x\sqrt{-1}})$$

$$= -\eta^2 + \tfrac{1}{2}\eta^4 - \tfrac{1}{3}\eta^6 + \&c.$$

$$-2\eta\cos x - \tfrac{1}{2}\,2\eta^2\cos 2x - \tfrac{1}{3}\,2\eta^3\cos 3x - \&c.$$

PROBLEMS.

368. *By Prof. J. Scheffer.*—In a quadrilateral $ABCD$, the diagonal AC makes with the sides the four angles $CAB = \alpha$, $ACB = \beta$, $ACD = \gamma$, $CAD = \delta$. Find the angles which the other diagonal BD makes with the sides.

369. *By R. J. Adcock.*—Show that the radius of curvature of an ellipse equals the cube of the radius vector divided by the rectangle of the semi axes; the radius vector being through the centre at right angles to the radius of curvature.

370. *By Prof. Edmonds.*—Divide a right angle into three parts α, β, γ, such that $(\cos\alpha) \div m = (\cos\beta) \div n = (\cos\gamma) \div p$.

371. *By Prof. E. B. Seitz.*—ACB is the quadrant of a circle, O the center of its inscribed circle; O_1, O_2, O_3, . . . O_n are the centers of a series of circles, each of which touches the preceding, the arc AB and the radius AC, the circle O_1 touches the circle O; and OH, O_1H_1, O_2H_2, . . . O_nH_n are the perpendiculars on AC, or the radii of the inscribed circles. If $AC=r$, $O_nH_n = x_n$, and $CH_n : O_nH_n = u_n$, prove that

$$u_n = \tfrac{1}{2}(\sqrt{2}+1)^{2n+1} - \tfrac{1}{2}(\sqrt{2}-1)^{2n+1},$$

$$x_n = \frac{2r}{2+(\sqrt{2}+1)^{2n+1}+(\sqrt{2}-1)^{2n+1}}.$$

372. *By William Hoover, A. M.*—A hemisphere, radius r, is resting with its convex surface on two planes, one perfectly smooth and inclined to the horizen at an angle α, the other being inclined at an angle β; if m be the coefficient of friction between the latter and the hemisphere, what is the position for rest?

373. *Selected by Prof. Eddy.*—Two particles of masses m and m' respectively, are connected by a string passing through a small fixed ring and are held so that the string is horizontal; their distances from the ring being a and a', they are let go. If ρ and ρ' be the initial radii of curvature of their paths, prove that

$$\frac{m}{\rho} = \frac{m'}{\rho'}, \text{ and } \frac{1}{\rho}+\frac{1}{\rho'} = \frac{1}{a}+\frac{1}{a'}.$$

374. *By R. S. Woodward.*—Prove 1st, that the probable error of any tabular value in a table of logarithms, trigonometric functions etc., is 0.25 of a unit of the last decimal place, supposing this place correct to the nearest unit; 2nd, that the average of the squares of probable errors of interpolated values depending on first differences only is $\frac{2}{3}(0.25)^2$.

ANNOUNCEMENT OF VOL. IX.—As this No. completes the 8th annual volume of the ANALYST, we are pleased to say to our readers that we have no thought of abandoning the publication, so long as we continue to receive the support and encouragement of the many able mathematicians who give character to our publication by their contributions to its pages.

The publication of the ANALYST was commenced with no exalted expectations of success, as the history of like publications in this country attests the difficulty of sustaining a periodical devoted exclusively to severe and exact scientific research.

It is therefore with some degree of gratification that we are able to state that the publication has been thus far conducted without pecuniary loss.